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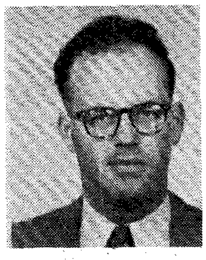

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Network Representation and Transverse Resonance for Layered Anisotropic Dielectric Waveguides

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Abstract—**First, the matrix wave impedance in an unbounded uniaxial lossless dielectric material is determined. Next, the transformation properties of the input impedance of a terminated anisotropic layer are established. It is then demonstrated that the boundary conditions in an anisotropic dielectric slab waveguide lead to a generalized transverse resonance condition involving the previously obtained matrix input impedances. Network equivalent representations are given for waveguides fabricated with dielectrics in polar and longitudinal orientations. The results show that a circuit approach to the analysis and design of planar anisotropic dielectric waveguides is feasible and practicable.**

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I. INTRODUCTION

THE CONCEPT of impedance and equivalent network representation is often used to obtain the dispersion characteristics of isotropic waveguides. As a result of the additional coupling mechanisms acting between field components in an anisotropic dielectric, the wave impedance expands into matrix form, and circuit equivalents are a great deal more cumbersome than those in the isotropic case. For this reason anisotropic layered waveguides are seldom treated by the methods of circuit analysis. Yet, there are some important configurations where the network approach provides both insight and a simple solution to the guidance problem.

This paper will investigate stratified waveguides fabricated with lossless uniaxial layers either in the polar or in the longitudinal configuration, as illustrated in Fig. 1. Polar configuration refers to a crystalline orientation where the optic axis lies in the plane of the interface whereas in the longitudinal case the optic axis is in the plane normal to the direction of propagation. The third uniaxial configuration, the equatorial, where the optic axis lies in the sagittal plane, leaves TE and TM modes uncoupled and, therefore, does not require special treatment.

The paper first discusses the coupled differential equation which determines the electromagnetic field distribution normal to the boundary interfaces. The solutions of this equation provide the matrix wave impedance of a homogeneous uniaxial dielectric. It is demonstrated that the matrix input impedance of terminated isotropic and anisotropic layers is subject to analogous transformation properties. By imposing the boundary conditions, the characteristic equation of various stratified guides fabricated with one or more anisotropic layers is obtained. In each case, it is found that the characteristic equation leads to a transverse resonance condition (TRC) [1] involving the previously determined matrix impedances. Utilizing equivalences existing between the matrix impedance and the matrix reflection coefficient, the generalized TRC is also expressed in terms of the latter. Network equivalents for the geometries analyzed in the text conclude the paper.

II. Matrix Wave Impedance

Consider a lossless stratified anisotropic waveguide consisting of homogeneous layers supporting wave propagation in the plane of the interfaces. If the interfaces are normal to x then the wavenumbers in the y and z directions k_y and k_z will be common throughout the entire waveguide. Consequently the electric field in a given layer can be expressed as

$$\mathcal{E}(x, y, z, t) = \bar{E}(x)\exp j(\omega t - k_y y - k_z z) \tag{1}$$

where $\bar{E}(x)$, the modal distribution, depends on the geometry and the material properties of the layer. As a result of the layered structure the Maxwell equations can be separated in each region into a set of coupled first-order linear differential equations involving only the transverse (y and z) field components, and into a set of algebraic equations relating the axial (x) and the transverse field components [2]. The differential equations are compactly expressed by

$$d\bar{f}(x)/dx = -jk_0 \boldsymbol{R}\bar{f}(x) \tag{2}$$

where $\bar{f}^T(x) = [E_y(x), H_z(x), E_z(x), -H_y(x)]$ is the transposed transverse field vector, $k_0^2 = \omega^2\epsilon_0\mu_0$,

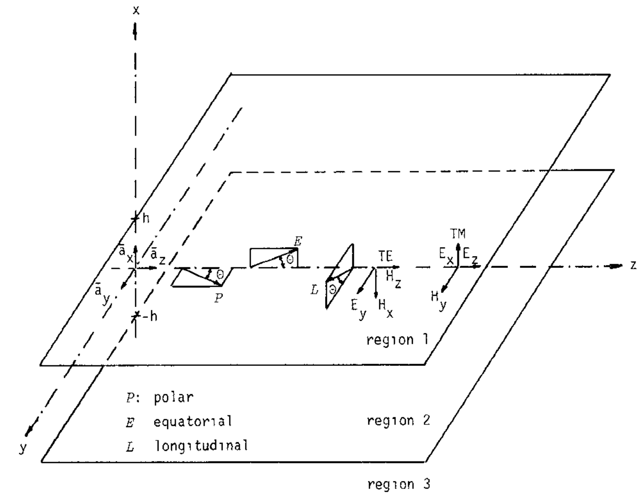

Fig. 1. Layered waveguide geometry. The optical axis orientation of a uniaxial dielectric in polar, equatorial, and longitudinal configuration is indicated. The electromagnetic wave propagates in the z direction.

is the coupling matrix, $\eta_0 = \sqrt{(\mu_0/\epsilon_0)}$,

$$\epsilon = \epsilon_0 \epsilon_r = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ & \epsilon_{yy} & \epsilon_{yz} \\ \text{sym.} & & \epsilon_{zz} \end{bmatrix} \tag{4}$$

is the symmetric real dielectric tensor, Δ_{ij} is the i, jth cofactor of ϵ_r and the propagation vector is denoted by $\bar{k} = k_0(\kappa\bar{a}_x + \xi\bar{a}_y + \beta\bar{a}_z)$. The algebraic equations relating the axial and transverse field components are expressed by a 2×4 matrix

$$\begin{bmatrix} E_x(x) \\ H_x(x) \end{bmatrix} = -\begin{bmatrix} \epsilon_{xy}/\epsilon_{xx} & \xi\eta_0/\epsilon_{xx} & \epsilon_{xz}/\epsilon_{xx} & \beta\eta_0/\epsilon_{xx} \\ \beta/\eta_0 & 0 & -\xi/\eta_0 & 0 \end{bmatrix} \cdot \bar{f}(x). \tag{5}$$

Of particular interest is the case depicted in Fig. 1, where one of the transverse coordinates is chosen to lie in the direction of propagation. This will be taken to be the z direction and let $\partial/\partial_y = -jk_y$ ($k_y = k_0\xi$) vanish throughout. The nonvanishing TE mode field components are then E_y, H_x, and H_z whereas those of the TM mode are E_x, E_z, and H_y as shown in Fig. 1. Coupling between TE and TM modes occurs only when there are nonzero elements in the off-diagonal blocks of $\boldsymbol{R}$. With ξ being zero only ϵ_{xy} and/or Δ_{yz} can contribute to such coupling.

The relative permittivity matrix appropriate to a particular configuration is obtained by rotating the principal coordinates of the anisotropic layer to coincide with the

$$\boldsymbol{R} = \begin{bmatrix} -\xi\epsilon_{xy}/\epsilon_{xx} & (1-\xi^2/\epsilon_{xx})\eta_0 & -\xi\epsilon_{xz}/\epsilon_{xx} & -\xi\beta\eta_0/\epsilon_{xx} \\ (\Delta_{zz}/\epsilon_{xx}-\beta^2)/\eta_0 & -\xi\epsilon_{xy}/\epsilon_{xx} & (\xi\beta-\Delta_{yz}/\epsilon_{xx})/\eta_0 & -\beta\epsilon_{xy}/\epsilon_{xx} \\ -\beta\epsilon_{xy}/\epsilon_{xx} & -\xi\beta\eta_0/\epsilon_{xx} & -\beta\epsilon_{xz}/\epsilon_{xx} & (1-\beta^2/\epsilon_{xx})\eta_0 \\ (\xi\beta-\Delta_{yz}/\epsilon_{xx})/\eta_0 & -\xi\epsilon_{xz}/\epsilon_{xx} & (\Delta_{yy}/\epsilon_{xx}-\xi^2)/\eta_0 & -\beta\epsilon_{xz}/\epsilon_{xx} \end{bmatrix} \tag{3}$$

device coordinates [3]. In the polar configuration, e.g.,

$$\epsilon_r = Q^T \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} Q$$

$$= \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 \cos^2\theta + \epsilon_3 \sin^2\theta & (\epsilon_3 - \epsilon_1)\sin\theta\cos\theta \\ 0 & (\epsilon_3 - \epsilon_1)\sin\theta\cos\theta & \epsilon_1 \sin^2\theta + \epsilon_3\cos^2\theta \end{bmatrix} \quad (6)$$

where θ is the angle between the optic axis of the uniaxial crystal and the z axis, and

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad (7)$$

is the orthogonal rotation matrix.

When the permittivity matrix is known, the coupling matrix, its four eigenvalues (the transverse wavenumbers κ_i, $i = 1$ to 4), and the characteristic equation ($|\cdot|$ denotes the determinant of a matrix)

$$|R - \kappa I| = 0 \quad (8)$$

can all be evaluated. In the absence of off-diagonal elements in the rotated ϵ_r, the characteristic equation contains only powers of β^2. Wave propagation in this case is, therefore, bidirectional. Furthermore, the characteristic equation contains only power of κ^2 and the eigenvalues of R can be conveniently sequenced so that κ_1 and $\kappa_2 = -\kappa_1$ pertain to the TE mode, while κ_3 and $\kappa_4 = -\kappa_3$ pertain to the TM mode. When ϵ_r is not diagonal, the eigenvalues of R generally are no longer pairs of the opposite sign; rather, they are a pair of real or conjugate complex numbers: $\kappa_{1,2} = \kappa_a \pm \kappa_b$ and $\kappa_{3,4} = \kappa_c \pm \kappa_d$. Therefore, the modal function in this case is

$$E_y(x') = [A_1 \exp(-j\kappa_b x') + A_2 \exp(j\kappa_b x')] \exp(-j\kappa_a x') + [A_3 \exp(-j\kappa_d x') + A_4 \exp(j\kappa_d x')] \exp(-j\kappa_c x') \quad (9)$$

where $x' = k_0 x$ is the normalized length in the x direction. Note that the terms in square brackets propagate in directions determined by the wave vectors $k_0(\beta\bar{a}_z + \kappa_a \bar{a}_x)$ and $k_0(\beta\bar{a}_z + \kappa_c \bar{a}_x)$, respectively. The amplitudes of the other three transverse field components can be obtained by solving for the eigenvectors of the coupling matrix [4].

In the longitudinal and polar configuration, the characteristic equation contains only powers of β^2 and κ^2. In these configurations, therefore, wave propagation is bidirectional and the transverse wavenumbers have pairwise opposite sign ($\kappa_2 = -\kappa_1, \kappa_4 = -\kappa_3$) even though all four transverse field components are coupled. The latter property can be viewed as a consequence of tr $(R) = 0$ which constrains

$$\sum_{i=1}^{4} \kappa_i = 0.$$

In the equatorial configuration, R is block diagonal permitting the existence of pure TE and TM modes. The characteristic equation in this case is a product of two factors: one arising from the determinant of the upper left block of R, characterizing TE modes, the second arising from the determinant of the lower right block of R, characterizing TM modes. The first factor is also a function of β^2 and κ^2; the second factor, however, includes a term in $\beta\kappa$ destroying both the bidirectional symmetry and the antisymmetry between κ_3 and κ_4.

An analysis of the Maxwell equations reveals that for a symmetric permittivity tensor, a scalar permeability, and a vanishing k_y, the modal distributions display the following parity (symmetry or antisymmetry in x) properties. In the longitudinal configuration where ϵ_{xy} is the sole nonzero off-diagonal matrix element of ϵ_r, E_x, E_y, H_x, H_z belong to the same parity group and E_z, H_z make up the group of opposite parity. In the polar configuration where $\epsilon_{yz} \neq 0$ the groups of opposite parity are E_y, E_z, H_x and E_x, H_y, H_z, respectively. In the equatorial configuration ($\epsilon_{xz} \neq 0$) for TE modes, E_y, H_x have the same parity opposite to that of H_z; while, the TM field components are neither purely symmetric nor antisymmetric. In the general case, where more than one off-diagonal element of ϵ_r is nonzero, pure symmetric or antisymmetric modes do not occur regardless of geometrical symmetry.

The paper concludes that in a symmetric uniaxial dielectric waveguide symmetric and antisymmetric modes

$$E_y^S(x') = A_1^S \cos\kappa_1 x' + A_3^S \cos\kappa_3 x'$$

and

$$E_y^A(x') = A_1^A \sin\kappa_1 x' + A_3^A \sin\kappa_3 x' \quad (10)$$

respectively, apply to all configurations except to equatorial TM modes where (10) is multiplied by the factor $\exp[j\beta(\epsilon_{xz}/\epsilon_{xx})x']$ resulting in angled wave propagation [5]. The angle enclosed between the positive z axis and the direction of wave propagation is $\alpha = \tan^{-1}(\epsilon_{xz}/\epsilon_{xx})$. The relative amplitudes H_z/E_y, E_z/E_y and H_y/E_y are essentially the eigenvectors of R and have been given by Wang and coworkers [6].

Having determined the field amplitudes and the transverse wave numbers, we can calculate the wave impedance of a homogeneous anisotropic material defined by

$$\bar{E}_\tau = Z(\bar{H}_\tau \times \bar{a}_x) \quad (11)$$

where $\bar{E}_\tau^T(x) = [E_y(x), E_z(x)]$ is the row vector of the transverse electric field and

$$Z = \begin{bmatrix} Z_{yy} & Z_{yz} \\ Z_{zy} & Z_{zz} \end{bmatrix} \quad (12)$$

is the wave impedance matrix of the dielectric. The elements of Z are found by substituting the modal field expressions into (11). Considering that the terms in the square brackets in (9) and the corresponding terms in the expressions of the other three field components must separately satisfy (11), this process indeed provides the necessary four equations. The resulting impedances for the three orientations discussed above are

$$Z=\eta_0/(a^{-1}-b^{-1})\begin{bmatrix}(a\kappa_1)^{-1}-(b\kappa_3)^{-1} & \kappa_1^{-1}-\kappa_3^{-1}\\ \kappa_1^{-1}-\kappa_3^{-1} & a\kappa_1^{-1}-b\kappa_3^{-1}\end{bmatrix} \tag{13}$$

for the polar orientation with $a \triangleq -\tan\theta$, $b \triangleq \kappa_1^2/(\epsilon_1 \tan\theta)$, $\kappa_1=(\epsilon_1-\beta^2)^{1/2}$, $\kappa_3=(\epsilon_3-\beta^2\epsilon_{zz}/\epsilon_1)^{1/2}$, and $|Z|=\eta_0^2\kappa_1/\epsilon_1\kappa_3$

$$Z=\eta_0/(c\kappa_3-d\kappa_1)\begin{bmatrix}c-d & \kappa_1-\kappa_3\\ \kappa_3-\kappa_1 & \kappa_1\kappa_3(c^{-1}-d^{-1})\end{bmatrix} \tag{14}$$

for a longitudinal orientation with $c \triangleq -\epsilon_1/(\beta\tan\theta)$, $d \triangleq \beta\tan\theta$, $\kappa_1=(\epsilon_1-\beta^2)^{1/2}$, $\kappa_3=[(\epsilon_3-\beta^2)\epsilon_1/\epsilon_{xx}]^{1/2}$, and $|Z|=\eta_0^2(c\kappa_1-d\kappa_3)/[(d\kappa_1-c\kappa_3)cd]$ and, finally

$$Z=\eta_0\begin{bmatrix}(\epsilon_1-\beta^2)^{-1/2} & 0\\ 0 & [(\epsilon_{xx}-\beta^2)/\epsilon_1\epsilon_3]^{1/2}\end{bmatrix} \tag{15}$$

for the equatorial orientation. Notice that (13) is symmetric whereas (14) is antisymmetric[1] and that both are sums of singular matrices. Regarding the wave admittance matrix, it should be noted that it is not simply the inverse of Z, although the determinant of Y is indeed the inverse of the determinant of Z. The wave admittance matrix is defined by

$$\bar{H}_\tau = Y(\bar{a}_x \times \bar{E}_\tau). \tag{16}$$

Consequently,

$$Y=\sigma Z^{-1}\sigma^T=|Z|^{-1}\begin{bmatrix}Z_{yy} & Z_{zy}\\ Z_{yz} & Z_{zz}\end{bmatrix} \tag{17}$$

where

$$\sigma=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}.$$

III. Transverse Resonance

The TRC is often used to determine the dispersion equation of propagating modes in isotropic layered waveguides. This section will generalize this condition to layered waveguides where one or more regions contain homogeneous uniaxial dielectrics. First, the stratified geometry shown in Fig. 2(a), consisting of an isotropic film over an anisotropic substrate covered by an isotropic (air) overlay, shall be discussed [7]. This will be followed by an analysis of the symmetric waveguide illustrated in Fig. 2(b) where all three regions are either in the polar or in the longitudinal orientation. Equatorial dielectrics are disregarded because in this material TE and TM fields remain independent and essentially isotropic conditions prevail.

To facilitate the analysis and to familiarize the reader with the notation, listed below are the transverse electric field components in the three regions of Fig. 2(a) assuming

[1]The Z matrix will be called antisymmetric when $Z_{zy}=-Z_{yz}$.

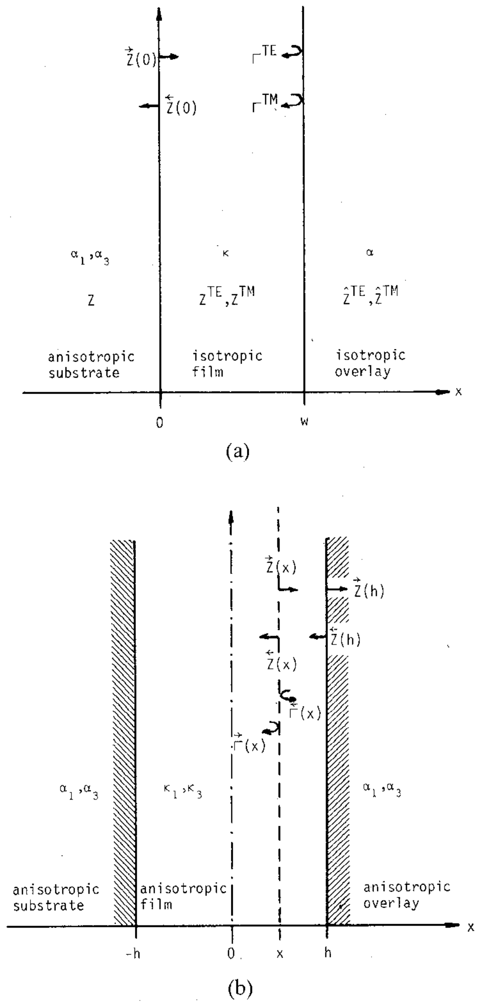

Fig. 2. The waveguide geometries analyzed in the text. In (a) the substrate is either polar or longitudinal. In (b) the waveguide is symmetric and the layers are either polar or longitudinal. The figure also indicates the transverse wavenumber(s) in each region and the reflection coefficients at selected constant x planes.

first a polar substrate:

$$\begin{aligned} E_y &= \hat{A}_1\exp(\alpha_1 x')+\hat{A}_3\exp(\alpha_3 x') \\ E_z &= \hat{a}\hat{A}_1\exp(\alpha_1 x')+\hat{b}\hat{A}_3\exp(\alpha_3 x') \end{aligned} \quad \text{substrate}$$

$$\begin{aligned} E_y &= A^{\mathrm{TE}}[\exp(-j\kappa x')+\Gamma^{\mathrm{TE}}\exp(j\kappa x')] \\ E_z &= A^{\mathrm{TM}}[\exp(-j\kappa x')-\Gamma^{\mathrm{TM}}\exp(j\kappa x')] \end{aligned} \quad \text{film}$$

$$\begin{aligned} E_y &= \hat{A}^{\mathrm{TE}}\exp[-\alpha(x'-w')] \\ E_z &= \hat{A}^{\mathrm{TM}}\exp[-\alpha(x'-w')] \end{aligned} \quad \text{cover} \tag{18}$$

where $\Gamma^{\mathrm{TE}}=\exp(j2\phi)$ and $\Gamma^{\mathrm{TM}}=\exp(j2\psi)$ denote TE and TM reflection coefficients, respectively, arising at the film–overlay interface. Let us further denote the local wave impedances by $Z^{\mathrm{TE}}=\eta_0/\kappa$, $\hat{Z}^{\mathrm{TE}}=j\eta_0/\alpha$, $Z^{\mathrm{TM}}=\eta_0\kappa$ and $\hat{Z}^{\mathrm{TM}}=-j\eta_0\alpha$. The boundary condition at $x=w$ ($w'=k_0w$) yields

$$\tan(\kappa w'+\phi)=jZ^{\mathrm{TE}}/\hat{Z}^{\mathrm{TE}} \tag{19}$$

for TE modes and

$$\tan(\kappa w'+\psi)=j\hat{Z}^{\mathrm{TM}}/Z^{\mathrm{TM}} \tag{20}$$

for TM modes. The boundary condition at $x=0$, on the other hand, results in the characteristic equation

$$\begin{bmatrix} 1 & 1 & -(1+\Gamma^{\mathrm{TE}}) & 0 \\ \alpha_1 & \alpha_3 & jy^{\mathrm{TE}}(1-\Gamma^{\mathrm{TE}}) & 0 \\ \hat{a} & \hat{b} & 0 & -(1-\Gamma^{\mathrm{TM}}) \\ \alpha_1/\hat{b} & \alpha_3/\hat{a} & 0 & -jy^{\mathrm{TM}}(1+\Gamma^{\mathrm{TM}}) \end{bmatrix} = 0 \tag{21}$$

where $y^{\mathrm{TE}}=\eta_0 Y^{\mathrm{TE}}$ is a normalized admittance. Recalling that the input impedances of TE and TM waves at $x=0$, as seen looking in the positive x direction indicated in Fig. 2(a), are

$$Z^{\mathrm{TE}}(0)=jZ^{\mathrm{TE}}\cot\phi \text{ and } Z^{\mathrm{TM}}(0)=-jZ^{\mathrm{TM}}\tan\psi \tag{22}$$

respectively, one can, after some algebra, recast (21) into what appears to be a generalized TRC:

$$|\overleftarrow{Z}(0)+\vec{Z}^{T}(0)|=0 \tag{23}$$

where

$$\vec{Z}(0)=\begin{bmatrix} Z^{\mathrm{TE}}(0) & 0 \\ 0 & Z^{\mathrm{TM}}(0) \end{bmatrix} \tag{24}$$

is the input impedance seen looking towards the right at $x=0$ and $\overleftarrow{Z}(0)$ is the wave impedance of the semi-infinite polar substrate given by (13), where a, b has been replaced by $\hat{a}$, $\hat{b}$, respectively, and, in accordance with the exponential decay of waves in this region, $\kappa_{1,3}$ has been replaced by $-j\alpha_{1,3}$. In the degenerate case when θ is either 0 or $\pi/2$, ϵ_{yz} vanishes, $\boldsymbol{R}$ becomes block diagonal, TE and TM modes decouple, and the off-diagonal elements of $\overleftarrow{Z}(0)$ become zero so that (23) reduces to

$$[\overleftarrow{Z}_{yy}(0)+Z^{\mathrm{TE}}(0)][\overleftarrow{Z}_{zz}(0)+Z^{\mathrm{TM}}(0)]=0. \tag{25}$$

Note that as long as one of the two matrices in (23) are symmetric, transposition of one has no effect because the value of the determinant remains unchanged by moving the T superscript from one matrix to another. Transposition becomes significant later when, in some instances, both impedances are antisymmetric.

When a longitudinally oriented substrate replaces the polar one the second expression of (18) changes to

$$E_z=j[(\alpha_1/\hat{d})\hat{A}_1\exp(\alpha_1 x')+(\alpha_3/\hat{c})\hat{A}_3\exp(\alpha_3 x')]$$

and the corresponding new characteristic equation is

$$\begin{bmatrix} 1 & 1 & -(1+\Gamma^{\mathrm{TE}}) & 0 \\ \alpha_1 & \alpha_3 & jy^{\mathrm{TE}}(1-\Gamma^{\mathrm{TE}}) & 0 \\ \hat{c} & \hat{d} & 0 & y^{\mathrm{TM}}(1+\Gamma^{\mathrm{TM}}) \\ \alpha_1/\hat{d} & \alpha_3/\hat{c} & 0 & j(1-\Gamma^{\mathrm{TM}}) \end{bmatrix} = 0. \tag{26}$$

The boundary condition at $x=w$ remains, of course, the same. Again (26) can be manipulated into the TRC (23) where now $\overleftarrow{Z}(0)$ represents the semi-infinite longitudinal substrate obtained from (14) by replacing c, d, and $\kappa_{1,3}$ with their corresponding parameters for the substrate region, $\hat{c}$, $\hat{d}$, and $-j\alpha_{1,3}$, respectively.

The paper then looks at the symmetric dielectric guides illustrated in Fig. 2(b) where all three regions are anisotropic, polar, or longitudinal. To simplify the algebra, it is useful to first determine the input impedance to an anisotropic layer of thickness h, terminated by a dissimilar anisotropic half space. Assuming the interface to be at $x=0$, we wish to find $\vec{Z}(-h)$.

Regardless of the orientation of the optical axis in the layer, E_y at $x=-h$ can always be chosen as $(h'=k_0 h)$

$$E_y(x'=-h')=A_1[\exp(-j\kappa_1 h')+\Gamma_1\exp(j\kappa_1 h')] + A_3[\exp(-j\kappa_3 h')+\Gamma_3\exp(j\kappa_3 h')] \tag{27}$$

where Γ_1 and Γ_3 are yet undetermined reflection coefficients at the interface. Having established the relationship between E_y and the other transverse field components in the previous section, we can write down H_z, E_z, and H_y corresponding to (27). In the polar configuration

$$E_z(x'=-h')=aA_1[\exp(-j\kappa_1 h')+\Gamma_1\exp(j\kappa_1 h')] + bA_3[\exp(-j\kappa_3 h')+\Gamma_3\exp(j\kappa_3 h')] \tag{28}$$

whereas, in the longitudinal case, the same field component is

$$E_z(x=-h')=(\kappa_1/d)A_1[\exp(-j\kappa_1 h')-\Gamma_1\exp(j\kappa_1 h')] + (\kappa_3/c)A_3[\exp(-j\kappa_3 h')-\Gamma_3\exp(j\kappa_3 h')]. \tag{29}$$

Substituting the field distributions into (11) and recalling that A_1 and A_3 are arbitrary, the four elements of the input impedance matrix can be computed. The results are identical to the wave impedance expressions given in (13) and (14) if κ_i is replaced by

$$\kappa_i'=\kappa_i[1-\Gamma_i\exp(-j2\kappa_i h')]/[1+\Gamma_i\exp(-j2\kappa_i h')], \quad i=1,3. \tag{30}$$

The material properties of the terminating half space relative to those of the front layer manifest themselves in Γ_1 and Γ_3.

Consider now a symmetric waveguide of $2h$ width, as shown in Fig. 2(b), bisected at $x=0$, supporting a symmetric $[E_y(x)=E_y(-x)]$ mode. Since the E_y must be maximum at the $x=0$ plane, the appropriate reflection coefficients are $\Gamma_i=1$ and, consequently, $\kappa_i'=j\kappa_i\tan(\kappa_i h')$, $i=1,3$. If the film material is in polar orientation, not only is E_y maximum at $x=0$ but so is E_z; thus they are both symmetric and the bisector can be regarded as an electrical open circuit. However, when the film is in longitudinal orientation, E_y and E_z have opposite parity: while the bisector is an open circuit in the y polarization, it acts as a short circuit in the z polarization. For antisymmetric modes $\Gamma_i=-1$, the modified wavenumber to use is $\kappa_i'=-j\kappa_i\cot(\kappa_i h')$ and the role of the bisector is reversed.

The boundary condition can be expressed in terms of impedances. Assuming polar dielectrics both in the film and in the external region, equating all four transverse field components at $x=h$ results in the determinantal equation

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \kappa_1' & \kappa_3' & j\alpha_1 & j\alpha_3 \\ a & b & \hat{a} & \hat{b} \\ \kappa_1'/b & \kappa_3'/a & j\alpha_1/\hat{b} & j\alpha_3/\hat{a} \end{vmatrix} = 0 \qquad (31)$$

where the expression appropriate to the parity of $E_y(x')$ must be substituted for κ_i'. When longitudinally oriented dielectrics are used, (31) must be modified by replacing a, b with c, d and $\hat{a}$, $\hat{b}$ with $\hat{c}$, $\hat{d}$, respectively. Algebraic manipulation again permits us to express (31) in the form of a TRC referred to $x=h$

$$|\overleftarrow{Z}(h)+\overrightarrow{Z}^T(h)|=0 \qquad (32)$$

where $\overrightarrow{Z}(h)$ is the wave impedance matrix of the homogeneous external region given by (13) or (14) ($\kappa_i=-j\alpha_i$, $i=1,3$) and $\overleftarrow{Z}(h)$ is the input impedance of the bisected film region given by the same expressions save for the $\kappa_i \to \kappa_i'$ modification. Although the boundary condition was applied at a conveniently chosen value of x, the boundary condition and the TRC must be satisfied at any arbitrary interface such as that shown dashed in Fig. 2(b).

When expressed in terms of input admittances the TRC assumes the same form as (32). This can be deduced from (17) by considering that for any 2×2 matrix $\boldsymbol{A}$, $\boldsymbol{\sigma A \sigma}^T=\boldsymbol{\sigma}^T\boldsymbol{A\sigma}=\boldsymbol{\sigma}^{-1}\boldsymbol{A\sigma}$, by further recalling that in general $|\boldsymbol{A}^{-1}+\boldsymbol{B}^{-1}|=|\boldsymbol{A}^{-1}||\boldsymbol{A}+\boldsymbol{B}||\boldsymbol{B}^{-1}|$ and by taking into account that neither $\overrightarrow{Z}$ nor $\overleftarrow{Z}$ is singular. An alternate form for the TRC is often expressed in the isotropic case as $1-\overleftarrow{\Gamma}\overrightarrow{\Gamma}=0$ [1, sec. 2.5] where $\overleftarrow{\Gamma}$ and $\overrightarrow{\Gamma}$ are "left" and "right" scalar reflection coefficients at an arbitrary interface within the guide, analogous to those reflection coefficients indicated in Fig. 2(b). When dealing with anisotropic waveguides, the reflection coefficient at a particular interface separating two dissimilar regions is defined by [6]

$$\bar{\boldsymbol{E}}_r=\boldsymbol{\Gamma}\bar{\boldsymbol{E}}_i \qquad (33)$$

where $\bar{\boldsymbol{E}}_r$ and $\bar{\boldsymbol{E}}_i$ are the reflected and incident transverse electric field vectors, respectively. Denoting the wave admittances of the left and right hand regions by $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, respectively, it can be shown [4] that

$$\boldsymbol{\Gamma}=\boldsymbol{\sigma}^T(\boldsymbol{Y}_1+\boldsymbol{Y}_2)^{-1}(\boldsymbol{Y}_1-\boldsymbol{Y}_2)\boldsymbol{\sigma} \qquad (34)$$

where $\boldsymbol{\sigma}$ has been defined earlier in connection with (17). Consider now the cross-sectional plane at x shown dashed in Fig. 2(b) and the reflection coefficient matrices referred to this plane $\overleftarrow{\Gamma}(x)$ and $\overrightarrow{\Gamma}(x)$. From (34), one can deduce that

$$\overrightarrow{\boldsymbol{Y}}(x)=\boldsymbol{Y}_f[\boldsymbol{I}-\boldsymbol{\sigma}^T\overrightarrow{\boldsymbol{\Gamma}}(x)\boldsymbol{\sigma}][\boldsymbol{I}+\boldsymbol{\sigma}^T\overrightarrow{\boldsymbol{\Gamma}}(x)\boldsymbol{\sigma}]^{-1} \qquad (35)$$

and

$$\overleftarrow{\boldsymbol{Y}}(x)=\boldsymbol{Y}_f[\boldsymbol{I}+\boldsymbol{\sigma}^T\overleftarrow{\boldsymbol{\Gamma}}(x)\boldsymbol{\sigma}]^{-1}[\boldsymbol{I}-\boldsymbol{\sigma}^T\overleftarrow{\boldsymbol{\Gamma}}(x)\boldsymbol{\sigma}] \qquad (36)$$

where $\boldsymbol{Y}_f$ is the characteristic impedance of the film material. Finally from (35) and (36), it can be shown [4] that the TRC in its generalized form as given in (32) is equivalent to

$$|\boldsymbol{I}-\overleftarrow{\boldsymbol{\Gamma}}(x)\overrightarrow{\boldsymbol{\Gamma}}(x)|=0 \quad \text{or} \quad |\overleftarrow{\boldsymbol{\Gamma}}(x)-\overrightarrow{\boldsymbol{\Gamma}}(x)|=0 \qquad (37)$$

depending on whether at least one impedance matrix is symmetric or both antisymmetric, respectively.

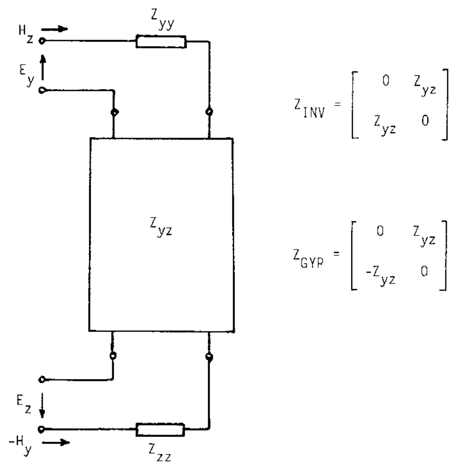

Fig. 3. A network equivalent of the wave impedance of polar and longitudinal dielectrics. The central two-port represents either an impedance inverter or a gyrator defined by the given impedance matrices, depending whether the dielectric is in polar or longitudinal configuration, respectively.

IV. Network Representations

A network representation of the matrix wave impedance of polar and longitudinal dielectrics given in (13) and (14) is shown in Fig. 3. The central two-port is an impedance inverter or a gyrator depending on whether the network represents a polar or a longitudinal dielectric material, respectively. Inasmuch as the $\boldsymbol{Z}$ matrix in (13) or (14) is a sum of two matrices, the equivalent circuit in Fig. 3 can be decomposed into two series connected two-ports.

An equivalent circuit of the layered waveguide illustrated in Fig. 2(a) appears in Fig. 4(a). The semi-infinite overlay occupying the $x>w$ region is represented by its TE and TM wave impedances, while the film region is equivalent to two uncoupled TE and TM transmission lines. The anisotropic substrate network model is the same as in Fig. 3, i.e., the central two-port assumes the role of either an impedance inverter or a gyrator as the case may be. $\overleftarrow{Z}_{yy}(0)$ and $\overleftarrow{Z}_{zz}(0)$ can be viewed as input impedances to infinitely long transmission lines extending in the negative x direction, supporting evanescent TE and TM waves.

Finally, the equivalent network of a symmetric anisotropic dielectric guide is given in Fig. 4(b). Here the circuit for the $x>h$ region is analogous to that of the $x<0$ region in Fig. 4(a). The circuit shown to the left of $x=h$ is equivalent to the half width dielectric film terminated at $x=0$ so that, for symmetric (antisymmetric) distributions in a polar material, both E_y and E_z are maximum (zero) and for the same distributions in a longitudinal material E_y is maximum (zero) while E_z is zero (maximum). To see that the network illustrated in Fig. 4(b) does indeed correspond to the TRC (32), view it as a two-port between ports 1–1′ and 2–2′, both ports short circuited. Then, by stipulating

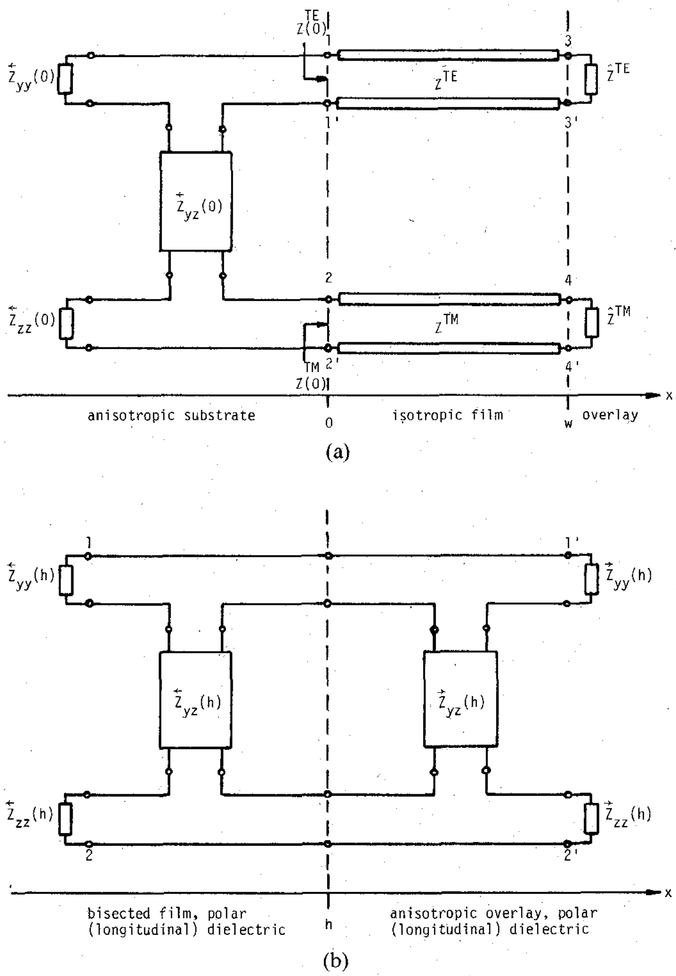

Fig. 4. Network equivalents of the layered waveguides illustrated in Fig. 2. In (a) the two-port on the left is either an impedance inverter or a gyrator depending whether the substrate dielectric is in the polar or longitudinal orientation respectively. In (b), illustrating the right half of Fig. 2(b), the above applies to both two-ports.

that the input impedance to this two-port (shorted at the output port 2–2′) be zero, one obtains condition (32).

V. Conclusion

The TRC has been generalized to include uniaxial dielectrics in the polar or in the longitudinal configuration. The equatorial configuration has been largely neglected because it fails to couple TE and TM waves. The TRC has been expressed in terms of impedances and admittances as well as in terms of reflection coefficients, all of which are in the present case 2×2 matrices.

Further work is in progress to include asymmetric waveguides and dielectrics of dissimilar configurations. The analysis is also being extended to cover lossy and magnetizable materials. Attention is focused on the fact that the wave impedances are composed of singular matrices. This property might considerably simplify the computation of the input impedance of multilayered structures.

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